Journal of Applied Mechanics and Technical Physics, Vol. 44, No. 2, pp. 198–203, 2003

MOTION OF A ROD IN A VISCOUS FLOW

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UDC 531.391.1:532.5.011

The equations of the dynamics of a finite-length curved rod in a viscous flow are derived. The longitudinal stability of the rod against small deflections from a rectilinear form is studied for two types of flow (pure and simple shear). The minimum flexural rigidity of the rod that ensures rod stability for any orientation in the flow is found. The effective viscosity of a suspension filled with rectilinear discrete fibers is estimated.

Key words: *rod, viscous fluid flow, stability, polymers.*

Shapovalov [1] showed that for a certain orientation of a filament in a viscous flow, the stretching force is zero. By the definition, a flexible filament transmits only tensile forces; therefore, the domain of applicability of the equations of [1] is limited by positive stresses in the filament. We can overcome this limitation by taking into account the flexural elasticity of the fiber, i.e., treating it as a rod.

Results of solution of the problem considered can be used to analyze flows of magnetorheological fluids with ellipsoidal particles and mixing of fiber-filled polymers. Flows of fiber-filled compositions have the following important features: increased flow pressure, variations in fiber distribution (orientation), and fiber breakage. Even if its matrix is a Newtonian fluid, a fiber-filled composition always exhibits non-Newtonian properties.

1. Equations of Dynamics. The following assumptions have been adopted for the problem considered. The inertial and gravitational forces are negligible. The rod is isolated both mechanically and hydrodynamically, i.e. it does not contact with other rods. The aeroelasticity effect is ignored and the rod does not introduce significant hydrodynamic perturbations into the fluid velocity fields. The flow is laminar and isothermal. Because the rod has a circular cross section, torsion does not occur upon flexure. The rod axis (elastic line) remains a flat curve, and the condition $\max(d/l, kd) \ll 1$ is satisfied (rod diameter d, rod length $2l$, and rod curvature k). The elastic strains caused by stretching or compression of the rod are ignored. The rod cross section is small compared with its length and remains unchanged under deformation, i.e., there is no pressure of longitudinal fibers. The rod cross sections (normal sections) remain plane under deformation (Bernoulli's hypothesis). Shears are ignored, transverse forces are determined from the equilibrium conditions, and strain equations are derived only for bending moments. The friction force acting on the rod is proportional to the relative flow rate.

Let us introduce a coordinate system (x, y, z) that is immovable in space or "frozen" in a fluid. The coordinates of the points on the elastic rod line s are denoted by x and y. The position of the curve s is described by the vector-function $\boldsymbol{r}(s,t)$, $-l \leqslant s \leqslant l$, where t is time. The x, y, and z directions form a right-hand oriented trihedron $(\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$ (see Fig. 1). Let $\boldsymbol{l}$ ($\boldsymbol{l} = \boldsymbol{r}_s$, $|\boldsymbol{l}| = 1$) denote a tangent vector to the elastic line, $\boldsymbol{n} = \boldsymbol{b} \times \boldsymbol{l}$ be a normal vector, and $\boldsymbol{b}$ be a unit vector parallel to the z axis.

According to [2], the equilibrium equations for the rod have the form

$$\boldsymbol{F}_s = -\boldsymbol{K}, \qquad \boldsymbol{M}_s = \boldsymbol{F} \times \boldsymbol{l}, \tag{1.1}$$

where $\boldsymbol{K} = A\boldsymbol{l}((\boldsymbol{V} - \boldsymbol{r}_t)\boldsymbol{l}) + B\boldsymbol{n}((\boldsymbol{V} - \boldsymbol{r}_t)\boldsymbol{n})$ is the linear density of external forces [1], $A = 2.1\pi\mu\sqrt{c}/\ln(0.952/\sqrt{c})$ is a coefficient that characterizes the longitudinal component of the friction force, μ is the fluid viscosity, c is the volumetric concentration of the fiber filler in the fluid, $B = 4\pi\mu/\ln(7.4/\mathrm{Re})$ is a coefficient that describes the transverse component of the friction force, $\mathrm{Re} = \langle v\rangle\rho d/\mu$ is the Reynolds number, ρ is the fluid density, and $\langle v\rangle$ is the characteristic velocity; the subscripts denote corresponding derivatives.

Volzhskii Polytechnical Institute at the Volgograd State Technical University, Volzhskii 404121. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 44, No. 2, pp. 56–62, March–April, 2003. Original article submitted July 2, 2002.

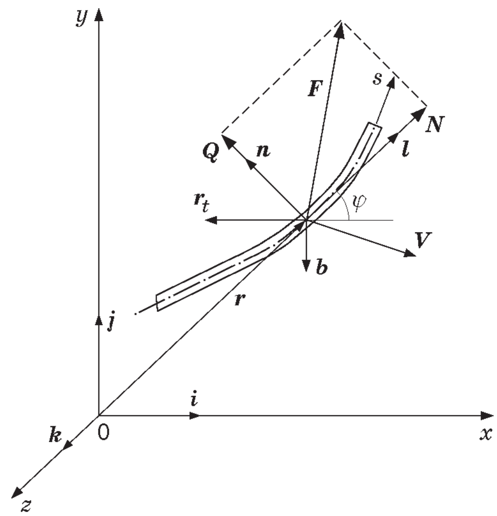

Fig. 1

With allowance for the relations $\boldsymbol{F} = (\boldsymbol{F}\boldsymbol{l})\boldsymbol{l} + (\boldsymbol{F}\boldsymbol{n})\boldsymbol{n} = N\boldsymbol{l} + Q\boldsymbol{n}$, $\boldsymbol{M} = M\boldsymbol{b}$, $\boldsymbol{l}_s = k\boldsymbol{n}$, $\boldsymbol{n}_s = -k\boldsymbol{l}$, $\boldsymbol{b}_s = 0$, $\boldsymbol{l} \times \boldsymbol{l} = 0$, and $\boldsymbol{n} \times \boldsymbol{l} = -\boldsymbol{b}$, Eqs. (1.1) become

$$(N_s - Q\varphi_s)\boldsymbol{l} + (N\varphi_s + Q_s)\boldsymbol{n} = -A\boldsymbol{l}((\boldsymbol{V} - \boldsymbol{r}_t)\boldsymbol{l}) - B\boldsymbol{n}((\boldsymbol{V} - \boldsymbol{r}_t)\boldsymbol{n}), \qquad M_s = -Q, \tag{1.2}$$

where $k = \varphi_s$ is the curvature of the rod, φ is the slope of the tangent line, N is the longitudinal force, Q is the shearing force, and M is the bending moment; $\boldsymbol{V} - \boldsymbol{r}_t = (v_x - x_t)\boldsymbol{i} + (v_y - y_t)\boldsymbol{j}$, $\boldsymbol{l} = \boldsymbol{i}\cos\varphi + \boldsymbol{j}\sin\varphi$, and $\boldsymbol{n} = -\boldsymbol{i}\sin\varphi + \boldsymbol{j}\cos\varphi$. Torsion is absent.

To close the problem, we use the condition of proportionality of the rod curvature to the internal-force moment [2]

$$M = EJ(\varphi_s - \varphi_{0,s}), \tag{1.3}$$

where E is the elastic modulus, $J = \pi d^4/64$ is the moment of inertia of the cross section, the value of $\varphi_{0,s}$ corresponds to the time $t = 0$, and $\varphi_0(s)$ is a function that describes the initial (natural) configuration of the rod and satisfies the conditions $\varphi_{0,s} = \varphi_{0,ss} = 0$ for $s = \pm l$.

The functions x, y, and φ are linked by the geometrical relations $x_s = \cos\varphi$ and $y_s = \sin\varphi$.

At the initial time, stresses are absent in the rod and forces and moments are absent at the free ends of the rod; therefore, the initial and boundary conditions of the problem are written as

$$t = 0,\ \boldsymbol{r} = \boldsymbol{r}_0\colon\quad \boldsymbol{M} = \boldsymbol{F} = 0; \qquad t > 0,\ s = \pm l\colon\quad \boldsymbol{F} = \boldsymbol{M} = 0, \tag{1.4}$$

where $\boldsymbol{r}_0 = x_0(s)\boldsymbol{i} + y_0(s)\boldsymbol{j}$ is the radius-vector at the initial time.

Solving the first equation in (1.2) for $\boldsymbol{V} - \boldsymbol{r}_t$ and differentiating both sides of the resulting relation with respect to s, we have the more convenient equation

$$\boldsymbol{V}_s - \boldsymbol{r}_{ts} = -B^{-1}[(N_s\varphi_s + N\varphi_{ss} + Q_{ss})\boldsymbol{n} + (N\varphi_s + Q_s)\boldsymbol{n}_s] - A^{-1}[(N_{ss} - Q_s\varphi_s - Q\varphi_{ss})\boldsymbol{l} + (N_s - Q\varphi_s)\boldsymbol{l}_s], \tag{1.5}$$

where $\boldsymbol{V}_s = (\boldsymbol{l}\nabla)\boldsymbol{V} = (((\boldsymbol{l}\nabla)\boldsymbol{V})\boldsymbol{n})\boldsymbol{n} + (((\boldsymbol{l}\nabla)\boldsymbol{V})\boldsymbol{l})\boldsymbol{l}$, $((\boldsymbol{l}\nabla)\boldsymbol{V})\boldsymbol{n} = \sin 2\varphi\,\partial v_y/\partial y - \sin^2\varphi\,\partial v_x/\partial y + \cos^2\varphi\,\partial v_y/\partial x$, and $((\boldsymbol{l}\nabla)\boldsymbol{V})\boldsymbol{l} = 0.5\sin 2\varphi(\partial v_x/\partial y + \partial v_y/\partial x) + \cos 2\varphi\,\partial v_x/\partial x$. Here the relations $\boldsymbol{n}_s = -\varphi_s\boldsymbol{l}$, $\boldsymbol{l}_s = \varphi_s\boldsymbol{n}$, $\boldsymbol{r}_{ts} = \varphi_t\boldsymbol{n}$, $\boldsymbol{b} \times \boldsymbol{b} = \boldsymbol{n} \times \boldsymbol{n} = 0$, $\boldsymbol{l} \times \boldsymbol{b} = -\boldsymbol{n}$, $\boldsymbol{l} = \boldsymbol{n} \times \boldsymbol{b}$, $\boldsymbol{r}_s = \boldsymbol{l}$, and $\nabla = (\partial/\partial x)\boldsymbol{i} + (\partial/\partial y)\boldsymbol{j}$ are taken into account.

According to (1.5), variations in the rod orientation or shape are due to the velocity gradient because the constant velocity component ($v_x = \text{const}$, $v_y = \text{const}$) causes a convective displacement of the rod along the corresponding coordinate axis without changing its configuration. Consequently, in studies of conformation transformations, one can place the origin of the Cartesian coordinate system at any point of the rod, for example, in the middle of the elastic axis ($x = 0$, $y = 0$, $s = 0$).

Thus, for the four unknown functions Q, N, M, and φ, we have the following equations: (1.3), (1.5), and the second equation in (1.2).

2. Stability Analysis. For the plane ($v_z = 0$) viscosimetric flows considered, the velocity components are written as follows: for pure shear, $v_x = g|\gamma|x$, $v_y = -g|\gamma|y$, and $g = \text{sign}\,\gamma$, and for simple shear, $v_x = g|\gamma_-|y$, $v_y = 0$, and $g = \text{sign}\,\gamma_-$ ($\gamma = \partial v_x/\partial x$ and $\gamma_- = \partial v_x/\partial y$ are the strain rates). To simplify the relations, we introduce a parameter g_1 that characterizes the flow type ($g_1 = 1$ for pure shear and $g_1 = 0$ for simple shear).

Let us introduce the following dimensionless variables and parameters: $\tau = t|\gamma g_1 + (1 - g_1)\gamma_-|$, $S = sl^{-1}$, $e = A/B$, and $K = EJ/[Bl^4|\gamma g_1 + (1 - g_1)\gamma_-|]$. The quantity $Al^2|\gamma g_1 + (1 - g_1)\gamma_-|$ is taken as the scale of the axial force N.

Excluding the functions M and Q from the second equation in (1.2) and Eqs. (1.3)–(1.5), we obtain the problem of the evolution of a curved rod in dimensionless form:

$$\varphi_\tau - (e+1)N_s\varphi_s - eN\varphi_{ss} + K\varphi_{ssss} - e^{-1}K\varphi_{ss}\varphi_s^2 = -g[g_1\sin 2\varphi + (1-g_1)\sin^2\varphi],$$

$$eN\varphi_s^2 - N_{ss} - K(1+e^{-1})\varphi_s\varphi_{sss} - Ke^{-1}\varphi_{ss}^2 = g[g_1\cos 2\varphi + 0.5(1-g_1)\sin 2\varphi], \tag{2.1}$$

$$\tau = 0,\ \varphi = \varphi_0(S):\quad \varphi_s = \varphi_{ss} = N = 0; \qquad \tau > 0,\ S = \pm 1:\quad N = \varphi_s = \varphi_{ss} = 0.$$

For a rectilinear rod ($d\varphi_0/dS = d\varphi/dS = 0$), an analytical solution of problem (2.1) is similar to that for a rectilinear filament [1]:

$$N_- = 0.5gg_1(1 - S^2)\cos 2\varphi_- + 0.25g(1 - g_1)(1 - S^2)\sin 2\varphi_-,$$

$$\varphi_- = g_1\arctan\,[\tan\,\varphi_0\exp\,(-2g\tau)] + (1 - g_1)\arctan\,[\tan\,\varphi_0/(1 + g\tau\tan\,\varphi_0)], \tag{2.2}$$

$$\varphi_{-,\tau} = -g_1 g\sin 2\varphi_- - (1 - g_1)g\sin^2\varphi_-.$$

Here and below, the subscript minus denotes the variables corresponding to a rectilinear rod.

The distributed load produced by viscous friction forces can generate both stretching and compressing forces in the flows in the rod, depending on the rod orientation [1]. Setting a small initial bending, we study the stability of the rod. Let us establish rod orientations for which perturbations will grow unboundedly or decay. We assume that the perturbations increase so rapidly that the evolution (turn along the stream) can be considered "frozen," i.e., the angle φ_- in the equations is treated as a parameter.

We introduce small perturbations of the rod shape α and the axial force β:

$$\varphi = \varphi_- + \alpha(S,\tau), \quad N = N_- + \beta(S,\tau), \quad \max\,(\alpha,\beta) \ll 1.$$

Linearization of Eqs. (2.1) yields the following equations for deflections:

$$\alpha_\tau + (e+1)SD\alpha_s - 0.5e(1 - S^2)D\alpha_{ss} + K\alpha_{ssss} = -2D\alpha,$$

$$-\beta_{ss} = g[-2g_1\sin 2\varphi_- + (1 - g_1)\cos 2\varphi_-]\alpha,$$

$$\tau = 0,\ \alpha = \alpha_0(S):\quad \alpha_s = \alpha_{ss} = \beta = 0; \qquad \tau > 0,\ S = \pm 1:\quad \alpha_s = \alpha_{ss} = \beta = 0,$$

where $D = g[g_1\cos 2\varphi_- + 0.5(1 - g_1)\sin 2\varphi_-]$.

The stability analysis reduces to an analysis of the first equation. We specify perturbations in the form $\alpha = A\exp\,(\lambda\tau)$, where $A(S)$ is an eigenfunction and λ is an eigenvalue. For the eigenfunction, we write the homogeneous problem

$$\lambda A + (e+1)SDA_s - 0.5e(1 - S^2)DA_{ss} + KA_{ssss} + 2DA = 0,$$

$$S = \pm 1, \qquad A_s = A_{ss} = 0.$$

An analysis of the solution for a complex plane shows that $\text{Im}\,\lambda = 0$, i.e., perturbations are absent. The eigenvalue problem was solved by Galerkin's method. As coordinate functions (approximations of the first two harmonics), we used the polynomials $15S - 10S^3 + 3S^5$ and $3S^2 - 3S^4 + S^6$ satisfying the boundary conditions. For the first two eigenvalues, we obtained the relations

$$\lambda_1 = -(0.572e + 1.19)g[2g_1\cos 2\varphi_- + (1 - g_1)\sin 2\varphi_-] - 34K,$$

$$\lambda_2 = -(5.345e + 1.336)g[2g_1\cos 2\varphi_- + (1 - g_1)\sin 2\varphi_-] - 146.77K. \tag{2.3}$$

For the position of the neutral equilibrium (for simple shear, $\varphi_- = 0$ and for pure shear, $\varphi_- = \pi/4$), the ratio of the eigenvalues $\lambda_2/\lambda_1 = 4.314$ obeys Eulerian stability theory, according to which, $\lambda_2/\lambda_1 = 4$ [3].

The first term in the relation for λ_1 in (2.3) characterizes the stability boundaries for a flexible filament (rod of zero rigidity, for which $K = 0$) [1]. The second term characterizes the effect of the bending rigidity of the rod on the stability boundaries. Accounting for the bending rigidity extends the stability region.

We note that there exists a critical rigidity of the rod K^* for which a rod of rigidity $K > K^*$ remains stable for any orientation. For pure shear ($g_1 = 1$) and with allowance for the equality $\max(-g\cos 2\varphi_-) = 1$, the critical rigidity is $K^* = 0.0336e + 0.07$. For simple shear ($g_1 = 0$) and with allowance for $\max(-g\sin 2\varphi_-) = 1$, the critical rigidity is half that for pure shear: $K^* = 0.0168e + 0.035$. Indeed, according to (2.2), the axial force for pure shear is twice that for simple shear.

Processing of fiber-filled polymers is characterized by intense dispersion of fibers and by the existence of a fracture limit for the filler can be reached. It can be assumed that fracture of high-module fibers (glass or carbon) proceeds by a mechanism of stability loss. For typical conditions of extrusion of glass-fiber polymers, the parameter values are as follows: $d = 10$ μm, $c = 0.05$, $\mu = 10^3$ Pa$\cdot$sec, $\mathrm{Re} = 1.2 \cdot 10^{-7}$, $e = 1.56$, $E = 7.5$ GPa, and $\gamma = 100$ sec^{-1}. In this case, the critical rigidity for pure shear $K^* = 0.122$ corresponds to a fiber length $2l = 0.3$ mm, which is in agreement with the study of [4–6], in which the fiber length after intense mechanical action was 0.1–0.9 mm. Under these conditions, the calculated [using formula (2.2) for $g = 1$, $S = 0$, and $\varphi_- = 0$] tensile stresses in the middle part of the fiber are $4N_-/(\pi d^2) = 59$ MPa, which is much smaller than fracture stresses under tension (3.5 GPa).

3. Viscosity of a Fiber-Filled Fluid. A fluid filled with suspended numerous fibers can be considered a homogeneous medium. The effective viscosity of this medium μ_+ differs from the viscosity of the main fluid (matrix) μ. We consider the case of small concentrations of suspended fibers, where the total amount of fibers is small compared with the fluid volume. In this case, is no contact and hydrodynamic effect among the fibers. The elastic axes of all fibers lie in the planes perpendicular to the z axis.

We determine the energy expenditures of the fluid flow due to the presence of a single fiber in the mixture (curved rod). The velocity of an individual point on the rod elastic axis is $\boldsymbol{r}_t$, and, hence, the relative fluid velocity is $\boldsymbol{V} - \boldsymbol{r}_t$; $\boldsymbol{K}$ is the local vector of the external force density. A force $\boldsymbol{K}\,ds$ acts on a rod of length ds. For a rod of length $2l$, the viscous friction energy W is given by the integral

$$W = \int_{-l}^{l} (\boldsymbol{V} - \boldsymbol{r}_t)\boldsymbol{K}\,ds.$$

With allowance for the first equation in (1.1) and the relations $\boldsymbol{V} - \boldsymbol{r}_t = -B^{-1}(N\varphi_s + Q_s)\boldsymbol{n} - A^{-1}(N_s - Q\varphi_s)\boldsymbol{l}$ and $\boldsymbol{F} = N\boldsymbol{l} + Q\boldsymbol{n}$, scalar multiplication yields the equation

$$W = \int_{-l}^{l} [B^{-1}(N\varphi_s + Q_s)^2 + A^{-1}(N_s - Q\varphi_s)^2]\,ds.$$

For a rectilinear rigid rod ($K > K^*$, $\varphi_s = 0$, and $Q = 0$), we have

$$W = A^{-1}\int_{-l}^{l} N_{-,s}^2\,ds.$$

The total number of fibers in the mixture is $2Vc/(\pi d^2 l)$ (V is the mixture volume); hence, the total energy expenditure W_Σ due to the flow around all fibers is given by the relation

$$W_\Sigma = 2WVc/(\pi d^2 l). \tag{3.1}$$

Taking into account Eq. (2.2), we obtain the following relation for simple shear ($g_1 = 0$):

$$W = (1/6)A\gamma_-^2 l^3 \sin^2 2\varphi_-. \tag{3.2}$$

Taking into account the additional energy expenditure due to the flow around the dispersed phase, the relation for the effective viscosity in simple shear is written as

$$\mu_+ = (\tau_{xy}\gamma_- V + W_\Sigma)/(\gamma_-^2 V), \tag{3.3}$$

where $\tau_{xy} = \mu\gamma_-$ is the tangential stress.

For a mixture filled with rectilinear fibers of identical length, diameter, and orientation, it follows from (3.1)–(3.3) that

$$\mu_+ = \mu\Big(1 + \frac{2.1}{3}\,\frac{l^2 c\sqrt{c}}{d^2 \ln(0.952/\sqrt{c})}\,\sin^2 2\varphi_-\Big).$$

The above relation implies that for $\varphi_- = 0$, the viscosity of the system is minimal, and for $\varphi_- = \pm\pi/4$, it is maximal. The fiber changes its orientation φ_- with time (synchronous rotation) in accordance with Eq. (2.2), and, hence, the effective viscosity also varies.

For polydisperse fibers whose fractions are all distributed uniformly over the volume of the medium, the viscosity is given by the relation

$$\mu_+ = \mu\Big(1 + \frac{2.1c\sqrt{c}}{3\ln(0.952/\sqrt{c})}\sum_{i=1}^{m}\frac{l_i^2}{d_i^2}\,\psi_i \sin^2 2\varphi_i\Big), \tag{3.4}$$

where $\varphi_i = \arctan\,[\tan\varphi_{0i}/(1 + g\tau\tan\varphi_{0i})]$, d_i, $2l_i$, and φ_{0i} are the diameter, length, and initial orientation of fibers of the ith fraction, respectively, m is the number of fractions, and ψ_i is the relative number of fibers of the ith fraction $\Big(\sum\limits_{i=1}^{m}\psi_i = 1\Big)$.

Because the neutral equilibrium position is unstable in simple shear, the fibers rotate with a nonuniform velocity [1]. Such instability may be caused by the fiber curvature, hydrodynamic perturbations generated by adjacent fibers, etc. If all fibers have the same orientation at the initial time, the effective viscosity approaches the constant value corresponding to the isotropic (chaotic) orientation under the law of damping oscillations. The region of neutral equilibrium can be extended if tensile strain is imposed on the main flow, which occurs, for example, on rollers.

We find the effective viscosity of the system for the isotropic (chaotic) orientation of fibers. According to Eq. (3.2), fibers with the orientations $+\varphi_-$ and $-\varphi_-$ correspond to the same value of W by virtue of the evenness of the function; therefore, we confine ourselves to the sector $0 < \varphi_- < \pi/2$. In the vicinity of the orientation φ_i, we distinguish a sector with the angle $\Delta\varphi = \pi/(2m)$ and treat l and d as functions of the angle $l(\varphi_i)$ and $d(\varphi_i)$. Taking into account the isotropy of $\psi = 2\Delta\varphi/\pi$, we can write the equality

$$\lim_{\substack{\Delta\varphi\to 0,\\ m\to\infty}}\sum_{i=1}^{m}\frac{l^2(\varphi_i)}{d^2(\varphi_i)}\,\frac{2\Delta\varphi}{\pi}\,\sin^2 2\varphi_i = \frac{2}{\pi}\int_0^{\pi/2}\frac{l^2(\varphi)}{d^2(\varphi)}\,\sin^2 2\varphi\,d\varphi.$$

For a one-fraction filler ($l = \text{const}$, $d = \text{const}$), the last integral is $0.5l^2/d^2$, and relation (3.4) becomes

$$\mu_+ = \mu\Big(1 + \frac{2.1}{6}\,\frac{l^2}{d^2}\,\frac{c\sqrt{c}}{\ln(0.952/\sqrt{c})}\Big).$$

Under pure shear ($g_1 = 1$), the total energy expenditure due to the flow around all fibers is found by formula (3.1) with allowance for (2.2):

$$W_\Sigma = [4Vcl^2/(3\pi d^2)]A\gamma^2\cos^2 2\varphi_-. \tag{3.5}$$

Since the tensile stress for pure shear is $\sigma_{xx} = 4\mu\gamma$ [7], the formula for the effective viscosity taking into account the energy expended in flowing around the particles has the form

$$\mu_+ = (\sigma_{xx}\gamma V + W_\Sigma)/(4\gamma^2 V).$$

With allowance for (3.5), we can write

$$\mu_+ = \mu\Big(1 + \frac{2.1}{3}\,\frac{l^2 c\sqrt{c}}{d^2\ln(0.952/\sqrt{c})}\,\cos^2 2\varphi_-\Big) \tag{3.6}$$

[function $\varphi_-(\tau)$ is determined in (2.2)]. All fibers have identical length, diameter, and orientation.

For m fractions, the relation for the effective viscosity is written as

$$\mu_+ = \mu\Big(1 + \frac{2.1c\sqrt{c}}{3\ln(0.952/\sqrt{c})}\sum_{i=1}^{m}\psi_i\,\frac{l_i^2}{d_i^2}\,\cos^2 2\varphi_i\Big), \tag{3.7}$$

where $\varphi_i = \arctan\,[\tan\varphi_{0i}\exp(-2g\tau)]$.

Regardless of the initial orientation of fibers, formulas (3.6) and (3.7) imply an asymptotic increase in viscosity with time because $\varphi \to 0$, $\varphi_i \to 0$, $\cos^2 2\varphi \to 1$, and $\cos^2 2\varphi_i \to 1$ as $\tau \to \infty$. The static equilibrium is stable. The maximum viscosity of the system is determined by the fiber concentration and the ratio $(l/d)^2$.

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